Questions

for *Spatial Vector Algebra*
by Roy Featherstone*

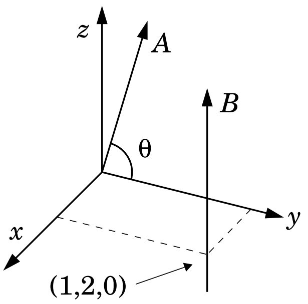

Figure 1

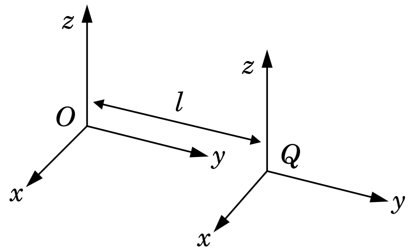

Figure 2

Question A1

Referring to Figure 1, work out the Plücker coordinates of the following spatial vectors:

(a) a unit rotational velocity about line A,

(b) a unit translational velocity in the direction of line A,

(c) a unit rotational velocity about line B,

(d) a translational velocity of 2 m/s in the direction of line B, and

(e) a twist velocity (i.e., a helical or screwing velocity) comprising a rotation of 2 rad/s and a translation of 1 m/s about and along line B.

Question A2

Figure 2 shows two parallel coordinate frames: $Oxyz$ and $Qxyz$, the latter being translated relative to the former by a distance l in the y direction. The Plücker basis associated with $Oxyz$ is $D_O = \{\boldsymbol{d}_{Ox}, \boldsymbol{d}_{Oy}, \boldsymbol{d}_{Oz}, \boldsymbol{d}_x, \boldsymbol{d}_y, \boldsymbol{d}_z\}$; the Plücker basis associated with $Qxyz$ is $D_Q = \{\boldsymbol{d}_{Qx}, \boldsymbol{d}_{Qy}, \boldsymbol{d}_{Qz}, \boldsymbol{d}_x, \boldsymbol{d}_y, \boldsymbol{d}_z\}$; and the coordinate systems defined by these two bases are called O and Q, respectively. The Plücker coordinates of a given spatial velocity vector $\hat{\boldsymbol{v}}$ are $\hat{\boldsymbol{v}}_O = [\omega_x\ \omega_y\ \omega_z\ v_{Ox}\ v_{Oy}\ v_{Oz}]^{\mathrm{T}}$ in O coordinates and $\hat{\boldsymbol{v}}_Q = [\omega_x\ \omega_y\ \omega_z\ v_{Qx}\ v_{Qy}\ v_{Qz}]^{\mathrm{T}}$ in Q coordinates.

(a) Express D_Q in terms of D_O.

(b) Express $\hat{\boldsymbol{v}}_Q$ in terms of $\hat{\boldsymbol{v}}_O$.

(c) Show that the expression $\omega_x \boldsymbol{d}_{Qx} + \omega_y \boldsymbol{d}_{Qy} + \cdots + v_{Qz} \boldsymbol{d}_z$ really is the same as $\omega_x \boldsymbol{d}_{Ox} + \omega_y \boldsymbol{d}_{Oy} + \cdots + v_{Oz} \boldsymbol{d}_z$.

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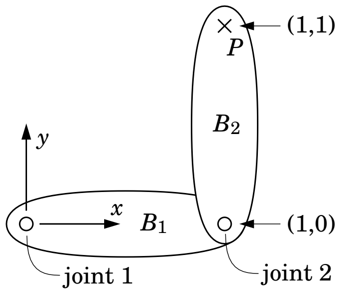

Figure 3

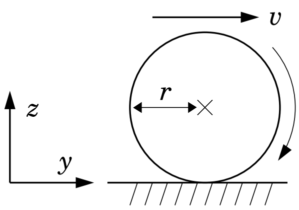

Figure 4

Question B1

Figure 3 shows a planar, two-link robot with two revolute joints. Each joint allows pure rotation of the distal link relative to the proximal link (or fixed base) about the joint's rotation axis. The axis of joint 1 coincides with the z axis; and the axis of joint 2 is parallel to the z axis but passes through the point $(1, 0)$ in the x–y plane. The two joint axes are represented by the motion vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, each being a unit rotation about the appropriate joint axis. The velocity of the distal body of joint i relative to the proximal body (or fixed base) is $\boldsymbol{s}_i \, \dot{q}_i$, where $\dot{q}_i$ is the joint's velocity variable. The two bodies, B_1 and B_2, have spatial velocities of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, respectively. Given this mechanism, work out the following:

(a) the Plücker coordinates of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$;

(b) the Plücker coordinates of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, expressed as a function of the joint velocity variables $\dot{q}_1$ and $\dot{q}_2$;

(c) the elements of the 6×2 Jacobian matrix, $\boldsymbol{J}$, that gives $\boldsymbol{v}_2$ as a function of the joint velocity vector $\dot{\boldsymbol{q}} = [\, \dot{q}_1 \; \dot{q}_2 \,]^{\mathrm{T}}$ (i.e., $\boldsymbol{v}_2 = \boldsymbol{J} \, \dot{\boldsymbol{q}}$); and

(d) the 3-D linear velocity of the point P from your answer to part (b).

Question B2

Suppose a pure force $\boldsymbol{f} = [\, -1 \; 0 \; 0 \,]^{\mathrm{T}}$ acts on B_2 in Figure 3 along a line passing through P.

(a) What are the Plücker coordinates of the forces that must be transmitted across the joints (i.e., the force transmitted from the base to B_1 and the force transmitted from B_1 to B_2) for the system to be in static equilibrium?

(b) What are the generalized forces at each joint for the system to be in static equilibrium? (Hint: the generalized force at joint i is $\tau_i = \boldsymbol{s}_i^{\mathrm{T}} \boldsymbol{f}_i$, where $\boldsymbol{f}_i$ is the spatial force across joint i.)

Question C1

Work out the Plücker coordinates of the accelerations $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ of the bodies B_1 and B_2 in Figure 3, expressed in terms of the joint velocity and acceleration variables $\dot{q}_1$, $\dot{q}_2$, $\ddot{q}_1$ and $\ddot{q}_2$.

Question C2

Figure 4 shows a cylinder that is rolling without slipping in the y direction on the x–y plane. The cylinder has a radius of r, and a constant forward velocity of v. Work out its spatial acceleration.

Question D1

Repeat the constrained-motion example on slides 50–51 without assuming that $\boldsymbol{v} = \boldsymbol{0}$.