An Introduction to Spatial (6D) Vectors

Questions Part 3

Topics: Plücker Coordinates, Differentiation and Acceleration
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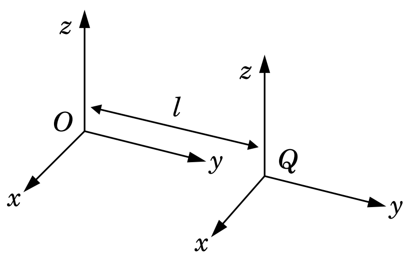

Figure 1

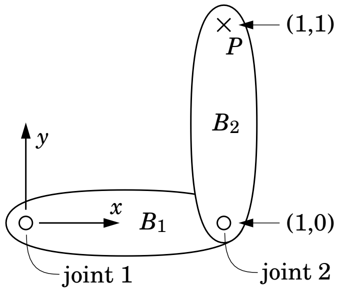

Figure 2

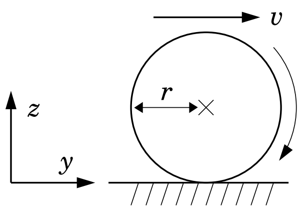

Figure 3

Question A1

Let $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ be elements of E^2 (the space of 2D Euclidean vectors), having Cartesian coordinates $[1\ \alpha]^{\mathrm{T}}$ and $[0\ 1]^{\mathrm{T}}$, respectively, where α is a scalar parameter. These two vectors are linearly independent, and therefore span E^2. So they can be used to form a basis $B = \{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ on E^2.

(a) Let $C = \{\boldsymbol{c}_1, \boldsymbol{c}_2\}$ be another basis on E^2. If C is reciprocal to B then what are the coordinates of $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$?

(b) Draw a picture showing the geometrical relationship between $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$.

Question A2

Figure 1 shows two parallel coordinate frames: $Oxyz$ and $Qxyz$, the latter being translated relative to the former by a distance l in the y direction. The Plücker basis associated with $Oxyz$ is $D_O = \{\boldsymbol{d}_{Ox}, \boldsymbol{d}_{Oy}, \boldsymbol{d}_{Oz}, \boldsymbol{d}_x, \boldsymbol{d}_y, \boldsymbol{d}_z\}$; the Plücker basis associated with $Qxyz$ is $D_Q = \{\boldsymbol{d}_{Qx}, \boldsymbol{d}_{Qy}, \boldsymbol{d}_{Qz}, \boldsymbol{d}_x, \boldsymbol{d}_y, \boldsymbol{d}_z\}$; and the coordinate systems defined by these two bases are called O and Q, respectively. The Plücker coordinates of a given spatial velocity vector $\hat{\boldsymbol{v}}$ are $\hat{\boldsymbol{v}}_O = [\omega_x\ \omega_y\ \omega_z\ v_{Ox}\ v_{Oy}\ v_{Oz}]^{\mathrm{T}}$ in O coordinates and $\hat{\boldsymbol{v}}_Q = [\omega_x\ \omega_y\ \omega_z\ v_{Qx}\ v_{Qy}\ v_{Qz}]^{\mathrm{T}}$ in Q coordinates.

(a) Express D_Q in terms of D_O. (As $\boldsymbol{d}_x$, $\boldsymbol{d}_y$ and $\boldsymbol{d}_z$ are the same in both bases, you have to find expressions only for $\boldsymbol{d}_{Qx}$, $\boldsymbol{d}_{Qy}$ and $\boldsymbol{d}_{Qz}$.)

(b) Express $\hat{\boldsymbol{v}}_Q$ in terms of $\hat{\boldsymbol{v}}_O$.

(c) Show that the expression $\omega_x \boldsymbol{d}_{Qx} + \omega_y \boldsymbol{d}_{Qy} + \cdots + v_{Qz} \boldsymbol{d}_z$ really is the same as $\omega_x \boldsymbol{d}_{Ox} + \omega_y \boldsymbol{d}_{Oy} + \cdots + v_{Oz} \boldsymbol{d}_z$.

Question B1

Figure 2 shows a planar, two-link robot with two revolute joints. It is the same robot that you encountered in Question B2 of Part 2. Each joint allows pure rotation of the distal link relative to the proximal link (or fixed base) about the joint's rotation axis. The axis of joint 1 coincides with the z axis; and the axis of joint 2 is parallel to the z axis but passes through the point $(1, 0)$ in the x–y plane. The two joint axes are represented by the motion vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, each being a unit rotation about the appropriate joint axis. The velocity of the distal body of joint i relative to the proximal body (or fixed base) is $\boldsymbol{s}_i \, \dot{q}_i$, where $\dot{q}_i$ is the joint's velocity variable. The two bodies, B_1 and B_2, have spatial velocities of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, respectively, and spatial accelerations of $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$, the latter being functions of the joint acceleration variables $\ddot{q}_1$ and $\ddot{q}_2$. In the earlier question you were asked to calculate the Plücker coordinates of $\boldsymbol{s}_1$, $\boldsymbol{s}_2$, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Now calculate the Plücker coordinates of $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Hint: bear in mind that $\boldsymbol{s}_1$ is fixed in space, but $\boldsymbol{s}_2$ is moving.

Question B2

Figure 3 shows a cylinder that is rolling without slipping in the y direction on the x–y plane. The cylinder has a radius of r, and a constant forward velocity of v. Work out its spatial acceleration.