An Introduction to Spatial (6D) Vectors

Questions Part 2

Topics: Motion and Force
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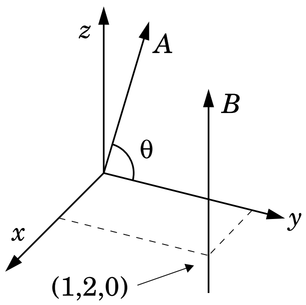

Figure 1

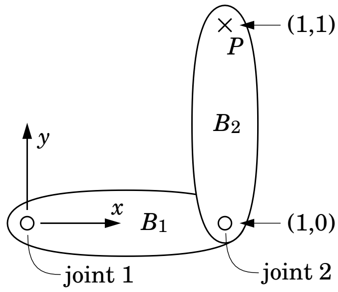

Figure 2

Question A1

Referring to Figure 1, work out the Plücker coordinates of the following spatial velocity vectors:

(a) a unit rotational velocity about line A (which lies in the y–z plane),

(b) a unit translational velocity in the direction of line A,

(c) a unit rotational velocity about line B,

(d) a translational velocity of 2 m/s in the direction of line B, and

(e) a twist velocity (i.e., a helical or screwing velocity) comprising a rotation of 2 rad/s and a translation of 1 m/s about and along line B.

Question A2

Suppose the spatial velocity of a rigid body has Plücker coordinates $[\omega_x\ \omega_y\ \omega_z\ v_{Ox}\ v_{Oy}\ v_{Oz}]^{\mathrm{T}}$. Find an expression for the vector field $\boldsymbol{V}(P)$ that maps the point P to the Euclidean velocity $\boldsymbol{v}_P$ of the body-fixed point at P. Express your answer in two ways:

(a) in terms of the vectors $\boldsymbol{\omega}$, $\boldsymbol{v}_O$ and $\overrightarrow{OP}$, and

(b) in terms of the Plücker coordinates, the coordinates P_x, P_y and P_z of P relative to O, and the basis vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

Question A3

At any given instant, a rigid body's velocity is, in general, a screwing motion, called a twist, which is characterized by an angular magnitude, a linear magnitude and a directed line in space that is the instantaneous screw axis. If the body's velocity is described by the two Euclidean vectors $\boldsymbol{\omega}$ and $\boldsymbol{v}_O$ then the angular magnitude is $|\boldsymbol{\omega}|$, which is the same thing as $\sqrt{\boldsymbol{\omega}\cdot\boldsymbol{\omega}}$. However, the linear magnitude is not $|\boldsymbol{v}_O|$. Assuming that $|\boldsymbol{\omega}| \neq 0$,

(a) find a formula for the linear magnitude of the twist; and

(b) find a formula for the pitch of the screwing motion, which is the linear magnitude divided by the angular magnitude.

Question A4

Continuing from question A3, the direction of the instantaneous screw axis is the same as the direction of $\boldsymbol{\omega}$; so if P is any one point on this line then the expression for a general point is

$$\overrightarrow{OQ}(s) = \overrightarrow{OP} + s\,\boldsymbol{\omega}$$

where s is an arbitrary scalar. So find an expression for any one point P on the instantaneous screw axis. (Hint: if P lies on the axis then $\boldsymbol{v}_P$ is parallel to $\boldsymbol{\omega}$. The answer includes at least one vector product.)

Question B1

Referring back to Figure 1, work out the Plücker coordinates of the following spatial vectors:

(a) a unit pure force acting along line A,

(b) a unit pure couple acting about line B, and

(c) a general wrench (a combination of a force and a couple) which is the sum of two unit pure forces, one acting along line A and the other along line B.

Question B2

Figure 2 shows a planar, two-link robot with two revolute joints. Each joint allows pure rotation of the distal link relative to the proximal link (or fixed base) about the joint's rotation axis. The axis of joint 1 coincides with the z axis; and the axis of joint 2 is parallel to the z axis but passes through the point $(1, 0)$ in the x–y plane. The two joint axes are represented by the motion vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, each being a unit rotation about the appropriate joint axis. The velocity of the distal body of joint i relative to the proximal body (or fixed base) is $\boldsymbol{s}_i\,\dot{q}_i$, where $\dot{q}_i$ is the joint's velocity variable. The two bodies, B_1 and B_2, have spatial velocities of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, respectively. Given this mechanism, work out the following:

(a) the Plücker coordinates of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$;

(b) the Plücker coordinates of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, expressed as a function of the joint velocity variables $\dot{q}_1$ and $\dot{q}_2$;

(c) the elements of the 6×2 Jacobian matrix, $\boldsymbol{J}$, that gives $\boldsymbol{v}_2$ as a function of the joint velocity vector $\dot{\boldsymbol{q}} = [\,\dot{q}_1\ \dot{q}_2\,]^{\mathrm{T}}$ (i.e., $\boldsymbol{v}_2 = \boldsymbol{J}\,\dot{\boldsymbol{q}}$); and

(d) the 3-D linear velocity of the point P from your answer to part (b).

Question B3

Suppose a pure force $\boldsymbol{f} = [\,-1\ 0\ 0\,]^{\mathrm{T}}$ acts on B_2 in Figure 2 along a line passing through P.

(a) What are the Plücker coordinates of the forces that must be transmitted across the joints (i.e., the force transmitted from the base to B_1 and the force transmitted from B_1 to B_2) for the system to be in static equilibrium?

(b) What are the torques at each joint for the system to be in static equilibrium? (Hint: the torque at joint i is $\tau_i = \boldsymbol{s}_i^{\mathrm{T}} \boldsymbol{f}_i$, where $\boldsymbol{f}_i$ is the spatial force across joint i.)