An Introduction to Spatial (6D) Vectors

Questions Part 4

Topics: Dynamics
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Question A1

Verify that the product of the spatial inertia $\boldsymbol{I}_O$, as shown on slide 7, with the velocity vector $\boldsymbol{v}_O = [\vec{\boldsymbol{\omega}}^{\mathrm{T}}\ \vec{\boldsymbol{v}}_O^{\mathrm{T}}]^{\mathrm{T}}$ is the same as the expression given for the momentum $\boldsymbol{h}_O$ on slide 5.

Question A2

What is the coordinate transformation rule for an inverse inertia?

Question A3

Suppose that two bodies, A and B, have inertias of $\boldsymbol{I}_A$ and $\boldsymbol{I}_B$, respectively, and they are initially at rest. Body A applies a force of $\boldsymbol{f}$ on body B; and so body B applies an equal and opposite force of $-\boldsymbol{f}$ back on body A (Newton's 3rd law). Find an expression for the relative inertia, $\boldsymbol{I}_{\mathrm{rel}}$, that relates the interaction force $\boldsymbol{f}$ to the relative acceleration of the two bodies, $\boldsymbol{a}_{\mathrm{rel}} = \boldsymbol{a}_B - \boldsymbol{a}_A$, according to $\boldsymbol{f} = \boldsymbol{I}_{\mathrm{rel}}\, \boldsymbol{a}_{\mathrm{rel}}$. Compare your result with the formula for $\boldsymbol{I}_{\mathrm{tot}}$ on slide 10. What do you notice?

Question A4

The spatial equation of motion on slide 11 incorporates both Newton's equation of motion for the centre of mass of a rigid body,† $\vec{\boldsymbol{f}} = m\ddot{\vec{\boldsymbol{c}}}$, and Euler's equation of motion for the rotational motion of a body about its centre of mass, $\vec{\boldsymbol{n}}_C = \bar{\boldsymbol{I}}_C\dot{\vec{\boldsymbol{\omega}}} + \vec{\boldsymbol{\omega}} \times \bar{\boldsymbol{I}}_C\vec{\boldsymbol{\omega}}$. Show that the spatial equation of motion does indeed incorporate these two equations.
Hint: Set $O = C$ so that $\vec{\boldsymbol{c}} = \boldsymbol{0}$ at the current instant.

Question B1

Redo the example on slides 18–23 (using $\boldsymbol{S}$) without the assumption $\boldsymbol{v} = \boldsymbol{0}$.

Question B2

Suppose we are told $\dot{\boldsymbol{S}} = \boldsymbol{v} \times \boldsymbol{S}$. Work out the values of $\boldsymbol{a}$ and $\dot{\boldsymbol{T}}^{\mathrm{T}}\boldsymbol{v}$ using the equations $\boldsymbol{v} = \boldsymbol{S}\boldsymbol{\alpha}$, $\boldsymbol{a} = \dot{\boldsymbol{v}}$ and $\boldsymbol{T}^{\mathrm{T}}\boldsymbol{S} = \boldsymbol{0}$.

Question B3

The equation of motion of a general rigid-body system can be expressed in the form $\boldsymbol{\tau} = \boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{C}$, where $\boldsymbol{\tau}$ and $\ddot{\boldsymbol{q}}$ are vectors of generalized forces and accelerations, $\boldsymbol{H}$ is a generalized inertia matrix and $\boldsymbol{C}$ is a generalized bias force vector. Express the equation of motion of the constrained rigid body (your answer to question B1) in this form, assuming $\ddot{\boldsymbol{q}} = \dot{\boldsymbol{\alpha}}$ and $\boldsymbol{\tau} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}$.

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†Strictly speaking, Newton's equation of motion applies to a particle. It was Euler who showed that this same equation applies also to the centre of mass of a rigid body.

Question B4

What is the kinetic energy of the constrained rigid body? Express it first as a function of $\boldsymbol{I}$ and $\boldsymbol{v}$, and then express it as a function of $\boldsymbol{H}$ and $\dot{\boldsymbol{q}}$ $(= \boldsymbol{\alpha})$.

Question C1

Develop recursive dynamics algorithms to calculate

(a) the total kinetic energy of a robot mechanism, and

(b) the total momentum of a robot mechanism.

Hint: Try adapting the recursive Newton-Euler algorithm.