An Introduction to Spatial (6D) Vectors

Questions Part 1

Topics: Vectors and Vector Fields
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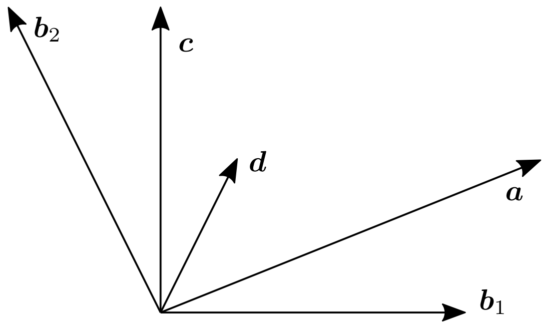

Figure 1

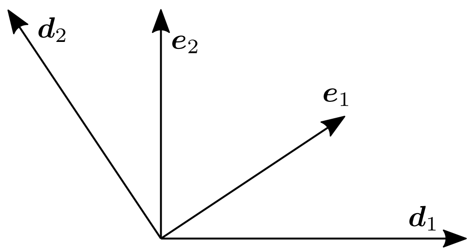

Figure 2

Question A1

Prove the statement on Slide 17 that $a_i = \boldsymbol{b}_i \cdot \boldsymbol{a}$.

Question A2

Given the definition of the cross product on Slide 9 and the definitions of the three basis vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ on Slide 19, work out the complete multiplication table for these three vectors.

Question A3

Prove the formula $\boldsymbol{a}\times\, \boldsymbol{b}\times = \boldsymbol{b}\boldsymbol{a}^{\mathrm{T}} - (\boldsymbol{a}\cdot\boldsymbol{b})\mathbf{1}_{3\times 3}$ on Slide 12. (Hint: look at slide 11.)

Question A4

Referring to Figure 1, what are the coordinates of the Euclidean vectors $\boldsymbol{a}$, $\boldsymbol{c}$ and $\boldsymbol{d}$ in the coordinate system defined by the basis $\{\boldsymbol{b}_1, \boldsymbol{b}_2\}$? (Hint: the coordinates are all integer multiples of 0.5.)

Question A5

Figure 2 shows the basis vectors of a dual coordinate system, which is an alternative to a Cartesian coordinate system. A dual coordinate system uses two bases: in this case, $\{\boldsymbol{d}_1, \boldsymbol{d}_2\}$ and $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$, which we shall call D and E, respectively. The basis vectors must satisfy

$$\boldsymbol{d}_i \cdot \boldsymbol{e}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

If $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ have coordinates $[1.25, 0]^{\mathrm{T}}$ and $[-0.6, 1]^{\mathrm{T}}$, respectively, then work out the coordinates of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$.

Question B1

Describe (in words) the following planar velocity fields:

(a) $\boldsymbol{d}_O + \boldsymbol{d}_x$,

(b) $2\boldsymbol{d}_O + \boldsymbol{d}_y$, and

(c) $\boldsymbol{d}_x + 2\boldsymbol{d}_y$.

Question B2

Suppose $\boldsymbol{V}$ is the vector field $\omega \boldsymbol{d}_O + r\omega \boldsymbol{d}_x$. Find an expression for the Euclidean vector $\boldsymbol{V}(P)$. (Hint: your answer will include the two basis vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ and the coordinates P_x and P_y of P.)

Question B3

Suppose $\boldsymbol{V}_1$ is a unit rotation about the origin (in a 2D plane) and $\boldsymbol{V}_2$ is a unit rotation about the point $(1, 0)$. What can you say about

(a) the vector field $\boldsymbol{V} = \alpha \boldsymbol{V}_1 + (1 - \alpha)\boldsymbol{V}_2$?

(b) the vector field $\boldsymbol{V}_1 - \boldsymbol{V}_2$?