

# An Introduction to Spatial (6D) Vectors and Their Use in Robot Dynamics

## Part 0: Introduction

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# What is a Spatial Vector?

*A spatial vector is a 6D vector that provides a complete description of the state of motion of a rigid body, or the forces acting upon it, in much the same way that a Euclidean vector provides a complete description of the state of motion of a particle, or the forces acting upon it.*

Euclidean vector      particle dynamics

spatial vector      rigid-body dynamics

# What is a Spatial Vector?

For Example . . .

equation of motion  
of a particle

$$\mathbf{f} = \frac{d}{dt} \mathbf{h}$$

Euclidean vectors

equation of motion  
of a rigid body

$$\mathbf{f} = \frac{d}{dt} \mathbf{h}$$

spatial vectors

In both cases, force is the rate of change of momentum; but the two equations use different kinds of vector.

# What is a Spatial Vector?

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*In particular, a spatial vector combines the linear and angular aspects of rigid-body motion or force into a single quantity.*

The purpose of this course is to explain how spatial vectors work, and how to use them.

# Why Are Spatial Vectors Useful?

They provide a concise notation for describing, analysing and calculating the kinematics and dynamics of individual rigid bodies and rigid-body systems.

- fewer quantities
- fewer equations
- less effort
- fewer mistakes

(For a worked example, see [sol3.pdf](#) and [sol6.pdf](#) in your notes.)

# What This Course Covers

- Vectors and Vector Fields
- Motion and Force
- Plücker Coordinates and Coordinate Transforms
- Differentiation and Acceleration
- Momentum, Inertia and the Equation of Motion
- Motion Constraints: D'Alembert's/Jourdain's Principle
- Robot Dynamics Example

# What You Should Do

If you want credits for taking this course:

- Learn the material.
- Pass the oral exam by the deadline (30th June).

If you do not want credits:

- Relax. Enjoy. Learn as much or as little as you wish.

# Checklist

- Are you using the 2022 notes (not an earlier year)?  
(<http://royfeatherstone.org/teaching/IntroSpVec2022.zip>)
- Do you have something to write on?
- Do you have something to write with?

Then you are ready to start.